

Colloquium Logicum 2026

Book of Abstracts

Tagung der Deutschen Vereinigung für Mathematische Logik
und für Grundlagenforschung der Exakten Wissenschaften (DVMLG)

Institute of Mathematics
Julius-Maximilians-Universität Würzburg

September 21–24, 2026

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Program Committee: Vasco Brattka, Anton Freund, Martin Hils, Benedikt Löwe (Chair),
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Local Organizing Committee: Antonia Dammrau, Giacomo Cozzi, Alexandra Fick,
Anton Freund (Chair), Dominik Wehr.

Monday

TIME	TALK	ROOM
13:15–14:00	Registration	?
14:00–15:10	Welcome Keynote Lecture: Awodey , <i>The Effective Infinity Topos</i> .	Zuse Hörsaal
15:10–15:15	<i>Coffee break</i>	
15:15–16:00	PhD Colloquium: Ludwig , <i>Model theory of difference fields with an additive character on the fixed field</i> .	Zuse Hörsaal
16:30–17:45	Contributed Talks section I Manca , <i>Fraïssé’s conjecture in systems of partial impredicativity</i> . Volpi , <i>Path Ramsey theorems and transitive colorings</i> . Vasung , <i>Computability of common fixed points of isometries</i> .	Zuse Hörsaal
16:30–17:45	Contributed Talks section II Bernieri , <i>The Copernican Theory of Properties and Propositions</i> . Hepburn , <i>Russell and Ramsey on Paradoxes Revisited</i> . Hofmann , <i>What is Potential? Ontological vs Ideological Potentialism</i> .	Hörsaal 4 (Naturwissenschaftlicher Hörsaalbau)

Tuesday

TIME	TALK	ROOM
09:15–10:10	Keynote lecture: Müller , <i>Infinite Games and Large Cardinals: Attacking Independence by Connecting the Hierarchies.</i>	Zuse Hörsaal
10:15–11:00	PhD Colloquium: Ketelsen , <i>Understanding the model theory of henselian valued fields by parts.</i>	Zuse Hörsaal
11:00–11:30	<i>Coffee break</i>	
11:30–12:45	Contributed Talks section I Engler , <i>Rigid Theories.</i> Szlufik , <i>On a gap in Scott spectra of $I\Sigma_n + \exp + \neg B\Sigma_n$.</i>	Übungsraum I
11:30–12:45	Contributed Talks section II Punčochář , <i>A type-theoretical approach to inquisitive semantics of mathematical questions.</i> Grigorian , <i>Graded Logical Pluralism: Category-Theoretic Approach.</i> Zeng , <i>A Jónsson-Tarski-Goldblatt Representation and Frame Definability for Noncontingency Logic.</i>	Übungsraum II
12:45–14:00	<i>Lunch break</i>	
14:00–15:15	Contributed Talks section I Friedmann , <i>Hans Blumenberg on 1966 “Artificial Intelligence” and Intermittent Mathematics.</i> Kant , <i>A perspectivist interpretation of deep disagreement in mathematics.</i>	Übungsraum I
14:00–15:15	Contributed Talks section II Meyer , <i>Finite Simple Groups in the primitive positive Constructions Poset.</i> Zhu , <i>Rank and independence of imaginaries in pairs of ACF.</i> Herrmann , <i>Expansion of Binary Cyclic Words.</i>	Übungsraum II
15:15–15:30	<i>Room change</i>	
15:30–16:15	PhD Colloquium: Osinski , <i>Interactions between Large Cardinals and Strong Logics.</i>	Zuse Hörsaal
16:15–16:45	<i>Coffee break</i>	
16:45–17:40	Keynote lecture: Kiefer , <i>Vertex Definability in Counting Logic.</i>	Zuse Hörsaal
19:30	Conference dinner	Weinstube Juliusspital

Wednesday

TIME	TALK	ROOM
09:15–10:10	Keynote Lecture: Rohr , <i>TBA</i> .	Zuse Hörsaal
10:15–11:00	PhD Colloquium: Uftring , <i>Well-Ordering Principles across Reverse Mathematics</i> .	Zuse Hörsaal
11:00–11:30	<i>Coffee break</i>	
11:30–12:45	Contributed Talks section I Lenz , <i>Large cardinals and abstract logics</i> . Lücke , <i>Hyperextracting cardinals</i> . Levine , <i>On Determinacy for Cut and Choose Games of Uncountable Length</i> .	Übungsraum I
11:30–12:45	Contributed Talks section II Wiesnet , <i>On the Limits of Recursive Characterizations in the Refined A-Translation</i> . Buriola , <i>On the Decidability of Languages for Differentiable Real Functions</i> . La Rosa , <i>Conservative Extensions of Intuitionistic Logic with Epsilon Terms over Predicate Abstraction</i> .	Übungsraum II
12:45–14:00	<i>Lunch break</i>	
14:00–15:15	Contributed Talks section I Neri , <i>Extracting bounds from proofs involving ultraproducts</i> . Schuster , <i>Ultimate Glivenko?</i> . Gaßner , <i>Abstract Computation over First-Order Structures: From configurations to first-order formulas</i> .	Übungsraum I
14:00–15:15	Contributed Talks section II Adlešić , <i>Signature extension of Quine's set theory with urelements</i> . Ihlenfeldt , <i>The consistency of New Foundations: What Holmes and Wilshaw did and what is not yet formalised in Lean</i> . Walzer , <i>A labelled sequent calculus for truthmaker logics</i> .	Übungsraum II
15:15–15:30	<i>Room change</i>	
15:30–16:25	Keynote Lecture: Maříková , .	Zuse Hörsaal
16:25–17:00	<i>Coffee break</i>	
17:00–17:55	Keynote Lecture: Pauly , <i>TBA</i> .	Zuse Hörsaal
18:15	DVMLG members' meeting	Zuse-Hörsaal

Thursday

TIME	TALK	ROOM
09:15–10:10	Keynote Lecture: Kish Bar-On , <i>Rethinking Mathematical Intuition, Agency and Practice in the Age of AI</i> .	Zuse Hörsaal
10:15–11:00	PhD Colloquium: Anttila , <i>Not Nothing: Nonemptiness in Team Semantics</i> .	Zuse Hörsaal
11:00–11:30	Coffee break	
11:30–12:40	Keynote Lecture: Aguilera , <i>TBA</i> . Closing Remarks	Zuse Hörsaal
12:40	Departure	

Abstracts of invited keynote speakers

JUAN AGUILERA (TU Wien, Austria), **TBA**.

TBA

STEVE AWODEY (Carnegie Mellon University, PA, USA), **The Effective Infinity Topos**.

As an elementary topos, Hyland’s effective topos $\mathcal{E}ff$ models extensional Martin-Löf type theory with an impredicative universe of propositions [1]. But it also contains another impredicative universe of so-called “modest sets” which is *not* a poset [2]—a fascinating combination not consistent with classical foundations. In joint work with Frey and Speight [3], we proposed a higher-dimensional version of this model with a *univalent* impredicative universe and used it to give new impredicative encodings of some (higher) inductive types. This model was based on cubical assemblies and exploited the constructive character of the recently introduced cubical Quillen model structures [4,6]. As was subsequently shown, however, the subtopos of 0-types in this model was not equivalent to $\mathcal{E}ff$, but to a larger realizability topos. In addition to containing a non-degenerate, impredicative, univalent universe, we believe that the elementary ∞ -topos $\mathcal{E}ff_\infty$ should include $\mathcal{E}ff$ as its subtopos of 0-types. This will also provide an example of a non-Grothendieck elementary ∞ -topos. As a candidate for $\mathcal{E}ff_\infty$ we here propose an ∞ -category of *coherent stacks* over the regular category of assemblies. We show that $\mathcal{E}ff_\infty$ is locally cartesian closed as an ∞ -category, that it is the ∞ -exact completion of the 1-category of partitioned assemblies, and that its subcategory of 0-types is indeed the effective 1-topos $\mathcal{E}ff$. This is joint work with Mathieu Anel and Reid Barton, building on prior joint work with Jacopo Emmenegger and Pino Rosolini [6].

- [1] J.M.E. Hyland, *The effective topos*. In: The L.E.J. Brouwer Centenary Symposium, A.S. Troelstra and D. van Dalen (eds.), Elsevier, 1982.
- [2] J.M.E. Hyland, *A small complete category*, Annals of Pure and Applied Logic, 40(2), pp. 135–165, 1988.
- [3] S. Awodey, J. Frey, S. Speight, *Impredicative encodings of (higher) inductive types*, LICS ’18, pp. 76–85, 2018.
- [4] C. Cohen, T. Coquand, S. Huber, A. Mörtberg, *Cubical Type Theory*. TYPES 2015, 2018.
- [5] S. Awodey, *Cartesian cubical model categories*, Lecture Notes in Mathematics, vol. 2385, Springer, 2025.
- [6] S. Awodey and J. Emmenegger, *Toward the effective 2-topos*, Mathematical Structures in Computer Science, vol. 35, 2025.

SANDRA KIEFER (University of Oxford, UK), **Vertex Definability in Counting Logic**.

Colour Refinement is a combinatorial algorithm that computes a vertex colouring for an input graph to reveal its structural asymmetries. It has a precise correspondence with the 2-variable fragment of the counting logic C. This connection has yielded a deep understanding of the graphs and relational structures that can be defined in the logic. In this talk, we shift the focus to definability of vertices. Those are vertices which can be “expressed” uniquely via a formula in the logic. They correspond to the vertices that receive a unique colour with respect to the algorithm. We show strong and tight lower bounds on vertex definability.

KATI KISH BAR-ON (Boston University & Harvard Business School, MA, USA), **Rethinking Mathematical Intuition, Agency and Practice in the Age of AI**.

The increasing integration of AI-based technologies into mathematical practice (such as AlphaGeometry, FunSearch, and Lean) poses a pressing philosophical challenge: how should we reconceive mathematical intuition, authorship, and agency in a landscape increasingly shaped by machine collaboration? This paper addresses that challenge by offering a historically and

philosophically grounded analysis of how AI systems are reshaping what it means to "do mathematics." Drawing from four recent case studies (AI-generated solutions in Euclidean geometry, knot theory, combinatorics, and the bunkbed conjecture (Trinh et al. 2024; Miao and Wang 2024; Davies et al., 2021)) we argue that AI programs are not merely tools for computation but co-constitutive agents that transform both the epistemic and social structures of mathematical practice. At the heart of this transformation is the changing status of the mathematician, a figure traditionally seen as the originator and sole author of mathematical knowledge. As AI-based programs generate solutions that are often beyond human anticipation, mathematicians find themselves collaborating with these technologies. While collaborations between mathematicians and computational tools are not entirely new, the current generation of AI technologies plays a more active role by generating original methods such as proofs, solutions, or examples, and thereby becoming integral to the creative process of producing new mathematical knowledge. This shift leads to a role reversal in how mathematical intuition is to be understood: from a capacity that may lead mathematicians to new proofs and theorems, it becomes retroactive – something that now needs to be interpreted rather than discovered. We examine the evolving notions of mathematical intuition historically, from Kant, Hausdorff, and Poincaré to more recent contributions by Lakatos, De Toffoli, Giardino, and Chudnoff. In doing so, we ask whether the unique human qualities that have historically characterized mathematical developments will remain characteristics of mathematical activity, or whether they will be supplemented or augmented in new ways or even shifted entirely by these new technologies. Most discussions on agency in mathematical practice refer to mathematical activity as human activity, focusing on what people do when they do mathematics (Ferreirós 2016; Hamami 2023), with little substantive exploration of how AI systems that generate original and creative proofs fit into this framework. However, interactions with AI-based programs require mathematicians to develop new skills and adapt to new modes of working. In this new kind of mathematical practice, mathematicians focus on developing and refining AI systems or specialize in interpreting and applying their outputs. The “working mathematician,” itself another concept in need of revision, is no longer just an individual or a group of mathematicians; instead, mathematical labor is increasingly distributed across human and machine actors, challenging the long-held view that mathematical knowledge is grounded in elementary human activities (Mancosu et al. 2005; De Toffoli and Giardino 2014; Chemla 2015). We conclude by situating our analysis within a broader historical trajectory, arguing that AI does not abolish intuition or agency but repositions them as relational properties of human–machine constellations, thereby reshaping the role of the mathematician and pointing to wider implications for philosophy and science.

- [1] Chemla K. (2015). Proof, generality and the prescription of mathematical action: a nanohistorical approach to communication. *Centaurus* 57(4):278–300.
- [2] Chudnoff, Elijah (2014). Intuition in Mathematics. In Linda Osbeck & Barbara Held (eds.), *Rational Intuition*. Cambridge University Press.
- [3] Davies A, Veličković P, Buesing L, Blackwell S, Zheng D, Tomašev N, Tanburn R, Battaglia P, Blundell C, Juhász A, Lackenby M, Williamson G, Hassabis D, Kohli P. (2021). Advancing mathematics by guiding human intuition with AI. *Nature* 600(7887): 70–74.
- [4] De Toffoli, S., & Giardino, V. (2014). An inquiry into the practice of proving in low-dimensional topology. *Boston Studies in the History and the Philosophy of Science*, 308, 315–336.
- [5] Ferreirós J. (2016). *Mathematical knowledge and the interplay of practices*. Princeton University Press, Princeton.
- [6] Hamami, Y. (2023). Agency in Mathematical Practice. In: Sriraman, B. (eds) *Handbook of the History and Philosophy of Mathematical Practice*. Springer, Cham. https://doi.org/10.1007/978-3-030-19071-2_48-1
- [7] Lakatos, Imre (1976) *Proofs and refutations: The logic of mathematical discovery*. Edited by J. Worrall and E. Zahar. Cambridge, UK: Cambridge University Press.
- [8] Mancosu P., Jørgensen K. F., Pedersen S. A. (eds) (2005) *Visualization, explanation and reasoning styles in mathematics*. Springer, Dordrecht.

- [9] Miao, Q., Wang, F. Y. (2024). *Artificial Intelligence for Science (AI4S)*. SpringerBriefs in Service Science. Springer, Cham.
- [10] Poincaré, H. (1902). *La Science et l'hypothèse*. Paris: Flammarion.
- [11] Trinh, Trieu H., Wu, Yuhuai, Le, Quoc V. et al. (2024). Solving olympiad geometry without human demonstrations. *Nature* 625, 476–482. <https://doi.org/10.1038/s41586-023-06747-5>

JANA MAŘÍKOVÁ (University of Vienna, Austria), **Geometric measures on definable sets.**

In the area of finitely additive measures and paradoxical decompositions, a standard situation is that of a group G of bijections of a space X acting on (a boolean algebra of) subsets of X , and the question is whether there is a G -invariant finitely additive probability measure on the (boolean algebra of) sets. If, for instance, G acts on itself by left translation, then this is just asking whether G is amenable in the classical sense, and there is a well-known connection between G being amenable and the existence of G -paradoxical subsets of X . In model theory, we start with a first-order structure M , and a group G of bijections of M^n , acting on a boolean algebra B of definable subsets of M^n . We wish to find a G -invariant measure on B . Usually, one will wish to put further restrictions on G , such as being M -definable, and there are again many well-understood instances of this situation. Here, we shall focus on the setting where G is a group generated by finitely many definable bijections (for reasons that will become clear). We survey known results, and discuss some applications and connections.

SANDRA MÜLLER (TU Wien, Austria), **Infinite Games and Large Cardinals: Attacking Independence by Connecting the Hierarchies.**

The independence phenomenon is a central theme in set theory. Moreover, starting from the Continuum Problem, there are numerous statements in various areas of mathematics that can neither be proven nor disproven in the usual axiomatic framework given by the Zermelo Fraenkel Axioms. The most promising approach to attack this issue is by studying extensions of the usual axiomatic framework, their connections, and their impact on previously independent statements. Two famous examples in set theory are the hierarchies given by the determinacy of infinite two-player games and large cardinals. The deep connection between these two hierarchies forms the backbone of inner model theory. We outline this connection and discuss recent progress as well as important open questions in the area.

ARNO PAULY (Swansea University, UK), **TBA.**

TBA

TABEA ROHR (University of Jena, Germany), **TBA.**

TBA

Abstracts of the PhD Colloquium

ALEKSI ANTILA (University of Padua, Italy), **Not Nothing: Nonemptiness in Team Semantics.**

We present various kinds of expressivity results in team semantics, centred around the study of bilateral state-based modal logic (BSML), a modal team logic introduced by Aloni to account for free-choice inferences. This logic extends classical modal logic with a nonemptiness atom NE which is true in a team just in case the team is nonempty. We first show that BSML is expressively complete for the class of all convex and union-closed modal team properties. We also introduce a convex variant of propositional dependence logic. Due to the failure of uniform substitution in team logics, one can distinguish multiple degrees to which a connective is definable in a team logic. We make these degrees precise, introduce accompanying senses in which one team logic may be said to extend another, and show that the convex variant of dependence logic extends dependence logic in a sense that corresponds to the existence of a compositional translation between the two. Finally, we examine the bilateral negation of BSML. We show that BSML is expressively complete for a certain notion of incompatibility or contradictoriness in that each pair of team properties incompatible in this sense is expressible by a BSML-formula together with its negation.

MARGARETE KETELSEN (MPI + RhFWU Bonn), **Understanding the model theory of henselian valued fields by parts.**

Model theory of henselian valued fields lies at the crossroads of algebra, number theory, and logic. In this talk, I present a modular approach to understanding the model theory of henselian valued fields by decomposing valuations into simpler components, studying these components individually, and then recombining the resulting information. This perspective leads to two key techniques: Standard decomposition and Composition AKE. Together, they provide a general framework for transferring model-theoretic properties between valued fields and their constituent parts. As an application, I will show how these methods yield a generalization of tilting to higher-rank valued fields.

STEFAN LUDWIG (University of Freiburg), **Model theory of difference fields with an additive character on the fixed field.**

A pseudofinite field is an infinite model of the common theory of all finite fields in the language of rings. In 1968, James Ax, building on number-theoretic results, gave an axiomatisation of the theory and proved its decidability, as well as a quantifier reduction. Hrushovski generalised this to the algebraic closures of finite fields equipped with a symbol for the Frobenius, and, more recently, to finite fields equipped with an additive character. In this talk I will give an introduction to the two main projects of my PhD thesis. The first, mainly motivated by model-theoretic questions, concerns the theory that arises when combining both of the above mentioned works of Hrushovski. The second, motivated by natural number-theoretic examples, concerns the model theory of (pseudo)finite fields equipped with both an additive and a multiplicative character.

JONATHAN OSINSKI (Czech Academy of Science), **Interactions between Large Cardinals and Strong Logics.**

Large cardinals are the yardstick for measuring the strength of strong mathematical theories. They often can be characterised in several fruitful ways, for instance through combinatorial properties or the existence of certain elementary embeddings. In this talk, we give an overview over the connections between large cardinals and model-theoretic properties of *strong logics*, i.e., of extensions of first-order logic. As it turns out, large cardinals all over the hierarchy are

characterised through their connections to model theory of strong logics, and several patterns in the large cardinal hierarchy arise in this way.

PATRICK UFTRING (UBW München), **Well-Ordering Principles across Reverse Mathematics.**

In this thesis, we study several well (partial) ordering principles and their strengths in the context of reverse mathematics over the weak base system RCA_0 : We begin with effective transfinite recursion, whose weak variant (due to Dzhafarov, Flood, Solomon, and Westrick) we show to characterize weak König's lemma and whose strong variant (due to Freund) we prove to be equivalent to arithmetical comprehension. Next, we consider sequences with gap condition and trees with weakly ascending labels, which can be used to characterize arithmetical transfinite recursion (extending results due to Gordeev) and whose maximal order types we determine precisely. Then, we study the termination of increasing Goodstein sequences, which we prove to characterize $\Pi_1^1\text{-CA}_0$ (following a conjecture due to Weiermann) using a close connection to 1-fixed points (a new concept by Freund and Rathjen). We finish with the uniform Kruskal theorem, whose equivalence to $\Pi_1^1\text{-CA}_0$ over CAC due to Freund, Rathjen, and Weiermann we improve to an equivalence over RCA_0 .

Abstracts of the contributed talks

TIN ADLEŠIĆ (University of Rijeka), **Signature extension of Quine’s set theory with urelements.**

In set theory, the introduction of new nonlogical (relation, function, and constant) symbols is usually a necessity and can be performed without any difficulties. The problem occurs when a theory imposes certain syntactic restrictions on formulas, such as stratifiability. While it is easy to say when a formula in the basic language is stratifiable, it becomes harder to do so when the formula contains other nonlogical symbols, introduced during theory development. In this talk, we present one approach based on signatures (tuples of type differences), which gives a uniform way to generalize stratification conditions and to prove that the notion of stratifiability remains unchanged when the new symbols are added.

JACOPO BERNERI (Oslo), **The Copernican Theory of Properties and Propositions.**

Building on ideas from Fine, Linnebo, and Roberts, this paper develops the *Copernican Theory of Properties and Propositions* (abbreviated with “CTPP”), a hyperintensional theory based on untyped λ -calculus that accommodates impredicative reasoning while avoiding paradoxes, such as Russell’s paradox and the Russell-Myhill. CTPP takes inspiration from the cumulative hierarchy of sets, except that here the ontology of properties and propositions is completely given, while to be constructed is their *ideology*: in CTPP we set up a process in which properties are iteratively *revealed*. The focus of the approach is specifically model-theoretical. We start with a base applicative structure, namely the standard model of the λ -calculus, and we use it to build consistent Copernican models, where the operator of application in the structure gets richer stage by stage. I argue that CTPP is a better approach than currently available theories of properties. With respect to nowadays popular higher-order approaches, the theory makes room for self-applicable and universal properties. Moreover, CTPP is more permissive and powerful than predicative theories, since it does not require restriction on quantification. Similarly, the theory is firmly classical, avoiding intuitionistic or trivalent semantics.

GABRIELE BURIOLA (Würzburg), **On the Decidability of Languages for Differentiable Real Functions.**

We study the decision problem for fragments of real analysis, building on Tarski’s decidability result for elementary real algebra — the first-order theory of the ordered field of reals, in which truth of any sentence is algorithmically verifiable. We investigate extensions of this theory that incorporate constructs about real functions while preserving decidability. We consider a family of languages $RDF^n (n \in \mathbb{N} \cup \{\infty\})$, each pertaining to C^n functions and progressively relaxing constraints on the use of the differentiation operator. The satisfiability problem for RDF^n is reduced to the truth problem for purely existential sentences of elementary real algebra. Using a suitable interpolation technique, decidability of RDF^2 has been established. Decidability of RDF^n for $n \geq 3$ and of RDF^∞ is conjectured, contingent on the construction of appropriate families of interpolating functions. A novel aspect of this work is a systematic investigation of the boundary between decidable and undecidable fragments, studying the sets $Sat_k(RDF^n)$ of formulas satisfiable by C^k functions, for $k, n \in \mathbb{N} \cup \{+\infty\}$.

MIRKO ENGLER (University of Vienna), **Rigid Theories.**

A traditional topic in model theory is the study of automorphisms of models, yielding insight into their structural properties. In this talk, we will present and investigate an analogous notion for theories: The relevant endomorphisms are relative self-interpretations of a theory, syntactic mappings which preserve theoremhood and logical structure. Self-interpretations of a theory T form a monoid under composition, and after quotienting by definable isomorphism, or homotopy, one obtains an invariant $End(T)$, which one can think of as measuring the freedom a theory

allows for when interpreting its axioms. Theories for which this invariant is trivial will be called *rigid*. After presenting the basic theory of (variants of) this notion, including some preservation results, we will examine a range of natural examples drawn from arithmetic, set theory, and algebra, including Presburger, Skolem, and Successor Arithmetic, as well as theories of groups and fields. Interestingly, Peano Arithmetic turns out to be non-rigid in general, but rigid with respect to retractions. However, a theorem due to Feferman and Scott can be used to show that True Arithmetic is rigid. In contrast, an iterated ultrapower construction due to Kanovei and Shelah shows that *ZFC* has no consistent rigid extension.

MICHAEL FRIEDMANN (Bonn), **Hans Blumenberg on 1966 “Artificial Intelligence” and Intermittent Mathematics.**

The German historian of ideas Hans Blumenberg discusses in the 1970s one of the earlier versions of “artificial intelligence”, namely the 1966 Weizenbaum’s ELIZA. In his analysis, Blumenberg criticizes the dream that reality’s and hence mathematics’ complexity can be captured through increasingly sophisticated formal systems. Blumenberg demands that any intelligence – artificial or not – should be “intermittent”, being one’s ability to interrupt itself and reflect. This reading of how ELIZA ‘behaves’ (or should behave) represents precisely what formal systems cannot replicate. With Blumenberg, I propose a different historical reading of what ELIZA may show us, also concerning the current developments in mathematics and logic with AI-based technologies. If one examines automatic proof assistants (e.g. Lean), which demand supplying a solid logical basis for future theorems to be proven, then Blumenberg’s critique shows that while such assistants can be extremely helpful, they cannot engage in the genuine self-reflection and in the understanding that mathematical practices require. Hence, do contemporary efforts at mathematical formalization thus remain blind not only to the rhetorical and intermittent nature of meaning that Blumenberg’s analysis of ELIZA revealed, but also to the practice of mathematics itself?

CHRISTINE GASSNER (Greifswald), **Abstract Computation over First-Order Structures: From configurations to first-order formulas.**

We provide a theoretical framework for linking various computational models, such as Turing machines, real RAMs, and decision trees. The focus is on BSS RAMs over arbitrary first-order structures. The programs of BSS RAMs can be defined syntactically and interpreted using first-order structures. At any given time, the overall state of a BSS RAM is a configuration that can be modified by a transition system. We introduce partial configurations to describe the execution of programs step by step at various levels of abstraction. The partial configurations of BSS RAMs can be generated by new transition systems, and the resulting sequences of partial configurations lead to suitable decision trees that can be evaluated using systems of first-order logic formulas.

ARNOLD GRIGORIAN (MCMP – LMU), **Graded Logical Pluralism: Category-Theoretic Approach.**

In this talk, I will try to argue for a graded pluralism grounded in categorical logic. The main idea behind that is that an “adequate logic” is one which is an internal language of a general class of categories (toposes, etc.). That is, the logic arises canonically as the internal language of an independently motivated class of categories widely used in mathematical practice. This criterion works with classical and intuitionistic logic, as well as other logical systems. In the talk, I will first argue for the categorical approach to logical pluralism in general. Then, I discuss ways in which this thesis can be precisely formulated and consider paraconsistent logics as a test case. I distinguish between weak paraconsistency, which can be considered categorically natural (in a bi-Heyting topos, after Lawvere), and strong paraconsistency, which has no canonical higher-order home. This relies on the fact that for a logic to be an internal language, the subobject

classifier of the category in question should have certain properties. The result is an asymmetry of canonicity, which points in the direction where our criterion cannot consider paraconsistent logic as a naturally arising categorical structure. I close by indicating how the same framework works for other logical systems, including HoTT as the internal language of higher toposes, extending the idea of what a logic can be from the categorical view.

JAMES HEPBURN (MCMP – LMU), **Russell and Ramsey on Paradoxes Revisited.**

Ramsey divided paradoxes into two types in his 1926 “The Foundations of Mathematics.” The first sort appears in presumably any formal language used to do mathematics or logic, whereas the second depends on the use of natural language or symbolism. Russell’s paradox falls in the first. Since the discovery of these paradoxes, many formal systems have been developed to prevent their appearance in mathematics, but it has been more intractable to determine what exactly gives rise to them. My talk will consider what exactly gives rise to Russell’s paradox using truthmaker semantics and will use these insights to consider if Ramsey’s division holds true, or if examples he gives actually fall into the other category. I will first observe that self predication itself results in an infinite regress of contained sets but does not always lead to a contradiction as the Paradox does. Second, I will argue that three assumptions produce Russell’s Paradox: Frege’s background assumption to Basic Law V stating there is a set for every predicate; that satisfaction of this predicate is not only sufficient, but necessary, for membership in its extension; and the Law of the Excluded Middle. I will then break down the steps taken to derive the contradiction and show where these assumptions enter, while using truthmaker semantics to clarify how set membership or its absence lead to the contradiction. I will conclude by using these insights to consider whether Ramsey’s division tracks all those paradoxes that are unavoidable in formal mathematics, or if there might be exceptions

BURGHARD HERRMANN (FU Berlin), **Expansion of Binary Cyclic Words.**

This talk is motivated by Turing’s major unfinished work, Morphogen Theory of Phyllotaxis, and recent biological observations. We describe the logic of cell division by deterministic and non-deterministic deductive systems on binary cyclic words and propose an application of the three-gap theorem.

DAVID HOFMANN (MCMP – LMU), **What is Potential? Ontological vs Ideological Potentialism.**

Contemporary debates in the philosophy of mathematics conceive of potentialism ontologically. Prominent potentialists, such as Øystein Linnebo and James Studd, argue for a modal dependence between sets and their elements, invoking Charles Parsons’s modal formulations of set theory to ground this ontological open-endedness. While Parsons argues for an ontological potentialism, in this talk we propose that Parsons also advocated for a second kind of potentialism: *ideological potentialism*. On this view, the base domain of mathematical objects (such as the natural numbers) can be understood as an actualized, extensionally definite totality. The indefinite extensibility resides not in the objects, but in the *ideology* – the open-ended hierarchy of predicates and classes we can dynamically formulate over those objects. To formally explicate this, the talk introduces a modalized version of Solomon Feferman’s Unfolding program. We demonstrate how Feferman’s substitution rule, when governed by an admissibility modality, models the generation of new predicates without expanding the underlying ontology. Ultimately, we argue that our work provides a formally precise and philosophically robust alternative version of potentialism.

COLIN IHLENFELDT (Hamburg), **The consistency of New Foundations: What Holmes and Wilshaw did and what is not yet formalised in Lean.**

In 1937, Quine proposed the set theory New Foundations (NF). In 2010, Randall Holmes announced a proof of the consistency of NF. This proof uses the equiconsistency of NF and Tangled Type Theory (TTT), proved by Holmes, and constructs a model of TTT inside ZFC. Starting in 2022, this construction was then formalized in Lean by Sky Wilshaw – the con-nf project. This talk gives an overview over the coarse structure of the proof and what was actually formalized. I then focus on the parts of the proof that have not yet been formalized, namely the equiconsistency of NF and TTT, and discuss a possible approach in closing the gap to achieve a full machine-checked proof of $Con(ZFC) \Rightarrow Con(NF)$.

DEBORAH KANT (Vrije Universiteit Brussel), **A perspectivist interpretation of deep disagreement in mathematics.**

This talk analyses three different kinds of disagreement in mathematics: putative proof disagreement, axiom disagreement, and philosophical disagreement. Only philosophical disagreement is classified as deep disagreement. Through two case studies—one concerning constructive mathematics and the other set-theoretic pluralism—I show that mathematicians can disagree over epistemic principles whose function is to justify mathematical axioms. I offer a perspectivist interpretation of deep disagreement by characterising an agent’s perspective as constituted by philosophical beliefs that are persistent, and play an instrumental role in the agent’s thought and action. The resulting framework is fallibilist, but also truth-compatible, and thus avoids radical pluralism. On this view, deep disagreement occurs when different perspectives clash. This explains why resolution is difficult and why remaining steadfast can be rational. At the same time, the view also entails a demand for openness among the agents involved, because an individual perspective may be false. Temporary shifts in perspective can render an alternative perspective more intelligible, may directly enrich the agent’s own thinking, and facilitate dialogue between parties involved in deep disagreement.

ELIO LA ROSA (MCMP – LMU), **Conservative Extensions of Intuitionistic Logic with Epsilon Terms over Predicate Abstraction.**

Extensions of Intuitionistic logic with Hilbert’s epsilon terms are famously non-conservative over Intuitionistic quantification. Partial solutions rely on weaker axioms, extensions to existence predicate or stronger intermediate logics, but do not scale well to theories. In this talk, I present a new approach, implicit in early papers by Melvin Fitting on Herbrand’s theorem for modal logics. Fitting extends quantified S4 modal logic with epsilon terms and predicate abstraction, using lambda binders to create a scoping mechanism for terms that mimics that of quantifiers. The resulting logic is conservative over S4. An immediate but overlooked consequence is its faithful embedding of Intuitionistic quantifiers over their modal and epsilon translation. I show that this result can be improved by internalizing the S4 modalities directly into lambda abstractors, thus eliminating modalities from the language entirely. The resulting logic is still conservative extension over Intuitionistic quantification, and remains so when adding identity and epsilon terms’ extensionality. Disjunction and Existence properties are also preserved. The logic can then be used as a base for developing constructive theories with choice.

ANNA LENZ (Hamburg), **Large cardinals and abstract logics.**

A cardinal κ is a *Jónsson* cardinal if every first order structure of size κ over a countable language has a proper elementary substructure of the same size. Aguilera, Bagaria and Lücke introduced in their 2024 paper "Large cardinals, structural reflection and the HOD conjecture" the notion of an exacting cardinal. These appeared to be singular in V but regular in HOD. Moreover, they showed that these very large cardinals are Jónsson in a stronger sense: For an exacting cardinal λ in any class $\mathcal{C}$ which consists of structures over a countable language and which is definable by a parameter in $V_\lambda \cup \{\lambda\}$, for every structure in $\mathcal{C}$ which has cardinality λ there exist a proper elementary sub- and a proper elementary superstructure of the same size in $\mathcal{C}$. The content of

this talk is joint work with Philipp Lücke. It is about a modification of the definition of exacting cardinals which is called weakly exacting. These cardinals will have some similar properties as exacting cardinals. Further more, we will introduce other notions of Jónssonness for abstract logics. One of them will turn out to be equivalent to being weakly exacting.

MAXWELL LEVINE (Freiburg), **On Determinacy for Cut and Choose Games of Uncountable Length.**

Many concepts in set theory are considered through the lens of infinite games. One particular genre goes at least back to work of Ulam and others in the 1960's, and is sometimes called the *cut and choose game*. We will discuss results on determinacy for variants of the cut and choose game, in particular instances of determinacy that are forced and instances of undetermined games which follow from certain assumptions. Letting $\mathcal{G}_{<\lambda}^{c\&c}(\mathbb{B})$ denote the version of the game where the choosing player is able to play at every stage before λ , we prove the consistency modulo large cardinals that the game $\mathcal{G}_{<\mu^+}^{c\&c}(\mathbb{B})$ is determined for all $\mathbb{B}$ and all cardinals μ . In particular, we answer a question of Zapletal and extend a theorem of Dobrinen.

PHILIPP LÜCKE (Hamburg), **Hyperexacting cardinals.**

Joint work with Juan Aguilera (TU Wien) and Joan Bagaria (Barcelona) introduced the notion of an exact cardinal as a candidate for a large cardinal assumption with far-reaching consequences for the structural behavior of ordinal definable sets. In this talk, I will review the basic properties of these cardinals and present results from work in progress with Juan Aguilera, which aims to determine how far this concept can be strengthened before reaching an inconsistent principle.

DAVIDE MANCA (TU Wien, Austria), **Fraïssé's conjecture in systems of partial impredicativity.**

Fraïssé's conjecture is the statement that countable linear orders form a well-quasi-order under the embeddability relation. The proof of that conjecture, due to Laver, is one of the most celebrated results in the theory of better-quasi-orders, a strengthening of the notion of well-quasi-order introduced by Nash-Williams. The theory of well and better-quasi-orders is of great interest to reverse mathematics, a branch of proof theory which aims to evaluate the strength of theorems from ordinary mathematics in terms of the set existence axioms needed to prove them. In particular, the strength of Laver's theorem is an important open problem in the field. On one hand, Shore showed that Fraïssé's conjecture implies arithmetical transfinite recursion. On the other hand, Montalbán obtained a proof which goes through in ATR_0 , save for the use of a certain infinitary Ramsey theorem, which is equivalent to $\Pi_1^1 - \text{CA}_0$. Due to its complexity, it is known that Laver's theorem itself cannot entail Π_1^1 comprehension. Freund identified a system of partial impredicativity in which Montalbán's proof goes through with small modifications, and that is properly weaker than $\Pi_1^1 - \text{CA}_0$. That system, called $\Pi_1^1 - \text{CA}_0^\Gamma$, can be defined in terms of the pseudo- β -reflection principles studied by Suzuki and Yokoyama. In joint work with Freund and Kowalik, we examine the strength of the system $\Pi_1^1 - \text{CA}_0^\Gamma$ with respect to ordinal analysis.

SEBASTIAN MEYER (TU Dresden), **Finite Simple Groups in the primitive positive Constructions Poset.**

Primitive positive constructions between relational structures are motivated by computational complexity of constraint satisfaction problems. They induce a partial order on equivalence classes of finite relational structures. The two topmost layers of the primitive positive constructability poset are known to consist of a single element each. In this talk, I show that this order has an infinite third layer, given by the equivalence class of the transitive tournament on three vertices (an oriented graph) and the equivalence classes of infinitely many permutation groups that are abstractly isomorphic to all the finite simple groups in a one-to-one correspondence and give an outlook on possible layers below. This talk is based by a joint work with Florian Starke.

MORENIKEJI NERI (TU Darmstadt), **Extracting bounds from proofs involving ultraproducts.**

An important application of ultraproducts in mathematics is the derivation of uniform bounds for existential statements via their validity in ultrapowers. More precisely, in the setting of first-order logic, one has the following uniformity principle:

Theorem: Let T be a first-order theory and $\{\varphi_n\}$ a collection of first-order formulas. Suppose that for all non-principal ultrafilters $\mathcal{U}$ and all first-order structures $\mathcal{M}$ with $\mathcal{M}_{\mathcal{U}} \models T$, we have: There exists $n \in \mathbb{N}$ such that $\mathcal{M}_{\mathcal{U}} \models \varphi_n$. Then there exists $N \in \mathbb{N}$ such that for all $\mathcal{M} \models T$ there exists $n \leq N$ with $\mathcal{M} \models \varphi_n$.

The proof of this result is inherently nonconstructive, which raises the natural question of whether bounds obtained in this way can be made explicit. This question lies close to the heart of proof mining [1], a program in mathematical logic, initiated by Kohlenbach and collaborators, that seeks to extract effective quantitative information from proofs in mainstream mathematics. Proof mining is underpinned by general proof-theoretic tools known as logical metatheorems, which guarantee that proofs carried out in suitable formal systems admit highly uniform computational content. These metatheorems not only provide algorithms for extracting this content, but also bounds their complexity in terms of the logical principles used. In case studies, most notably the work of Simmons and Towsner [2], it has been observed that analyzing proofs based on the uniformity principle through the lens of existing metatheorems of proof mining often yields explicit uniform bounds. Building on earlier work of Günzel and Kohlenbach [3], we present a new metatheorem that explains this phenomenon in a systematic way. As an application, we derive new explicit bounds for results in [4] which we obtained through this methodology. This is joint work with Ulrich Kohlenbach and Jin Wei.

- [1] U. Kohlenbach, *Applied Proof Theory: Proof Interpretations and their Use in Mathematics*, Springer Monographs in Mathematics. Springer Berlin, Heidelberg, 2008.
- [2] W. Simmons and H. Towsner, Proof mining and effective bounds in differential polynomial rings, *Advances in Mathematics*, 343:567–623, 2019.
- [3] D. Günzel and U. Kohlenbach, Logical metatheorems for abstract spaces axiomatized in positive bounded logic, *Advances in Mathematics*, 290:503–551, 2016.
- [4] Gabriel Conant, Anand Pillay, and Caroline Terry, A group version of stable regularity, *Mathematical Proceedings of the Cambridge Philosophical Society*, 168(2):405–413, 2020.

VÍT PUNČOCHÁŘ (Charles University Prague), **A type-theoretical approach to inquisitive semantics of mathematical questions.**

The most influential contemporary framework for the logical analysis of questions is inquisitive semantics, built on the idea that a question expresses an informational gap and that answering it means filling that gap. This analysis of questions faces a key limitation. In many cases of questioning, the questioner already possesses all the information needed to determine the answer; what is lacking is the ability to derive it. Mathematical questions are the clearest illustration: anyone who accepts the axioms of arithmetic already has all the relevant information, yet many arithmetical problems remain genuinely unsettled until a proof is found. In such cases, the deficit is inferential rather than informational. In my paper I present a type-theoretic approach to inquisitive semantics. It reveals that the contrast between the standard Tarskian semantics of statements and the inquisitive semantics of questions parallels the contrast between Church’s simple theory of types, in which propositions are captured at the level of terms, and Martin-Löf’s constructive type theory, which identifies propositions with types. I will argue that this type-theoretic perspective allows us to capture both the informational and the inferential dimensions of questioning.

PETER SCHUSTER (Università degli Studi di Verona), **Ultimate Glivenko?**

A simple candidate for a Glivenko master theorem applies not only to the customary generalisations, such as Kuroda’s to predicate logic, but always when an extension by axioms of a suitable

consequence relation is to be conservative over the Kleisli extension with respect to a given nucleus. For the generalisation of Glivenko's theorem to hold it is necessary and sufficient that the additional axioms are derivable in the Kleisli extension or, equivalently, belong to the Jacobson radical. This conservation criterion amounts to the nuclear shift for implication—a variant of Kuroda's double negation shift—whenever the additional axioms express stability, i.e. that the nucleus be identified with identity, as is the case not only for Glivenko's original theorem.

(Emerged from and based on joint work in various composition with Giulio Fellin, Daniel Misselbeck-Wessel and Sara Negri.)

PATRYK SZLUFIK (Warsaw), **On a gap in Scott spectra of $I\Sigma_n + exp + \neg B\Sigma_n$.**

For every countable structure M there is a sentence of $\mathcal{L}_{\omega_1, \omega}$ in the signature of M , which is satisfied by a countable structure N if and only if $N \simeq M$. This allows for introduction of various complexity measures of structures, e.g. via "quantifier complexity" of the least complex among such sentences, where Π_α stands for universal steps in the hierarchy (countable conjunctions [disjunctions] force universal [existential] steps). We say that Scott rank of a structure is α , if said complexity is $\Pi_{\alpha+1}$. Montalbán and Rossegger initiated Scott analysis of models of Peano Arithmetic, which was further extended in our joint work with David Gonzalez, Dino Rossegger and Mateusz Łęłyk. With Leszek Kołodziejczyk and Mateusz Łęłyk, we conducted similar analysis for subsystems of PA . Among others, we have shown that possible Scott ranks of models of $I\Sigma_n + \neg B\Sigma_{n+1}$ include $n+1$ and ordinals $\alpha \in (n+2, \omega_1)$ with the case $n+2$ unresolved. Building on recent results about coded ultrapowers and methods developed in Scott analysis of arithmetic, I construct models of $I\Sigma_n + \neg B\Sigma_{n+1}$ with Scott ranks $n+2$. We will explore the proof, emphasizing what can be used in the broader Scott analysis of weak arithmetics.

PATRIK VASUNG (Zagreb), **Computability of common fixed points of isometries.**

We study the computability of the set of all common fixed points of isometries for compact subsets of $\mathbb{R}^n$. We show that if X is a computable subset of $\mathbb{R}^n$, then the set of all common fixed points of isometries of X is semicomputable. We further show that, in general, this set is not computable, and that it may be nonempty while containing no computable points, even for computable subsets of $\mathbb{R}^n$. Finally, we prove that if X is computable and convex, then the set of all common fixed points of isometries of X is computable.

ANDREA VOLPI (Warsaw), **Path Ramsey theorems and transitive colorings.**

There is a proof that Ramsey's theorem for triples and 2 colors implies ACA that uses a weaker notion of homogeneity called path homogeneity. A set $H = \{h_0 < h_1 < \dots < \}$ is path homogeneous if for all i , the sets (h_i, h_{i+1}, h_{i+2}) take the same color. Starting from this observation, we study different notions of Ramsey-like statements depending on restrictions of the colorings considered and the notion of homogeneity required. Some of these notions are already well studied under different names. For instance the ascending descending principle ADS is exactly Ramsey's theorem for pairs and 2 colors restricted to transitive colorings and where solutions are homogeneous sets, while Erdős–Moser theorem EM is Ramsey's theorem for pairs and 2 colors with no restriction on the colorings considered and where solutions are infinite set where the coloring is transitive. We define and study generalizations of these and other principles for n -tuples where $n \geq 3$ and more than $k \geq 2$ colors in the framework of reverse mathematics. This is joint work with Lorenzo Carlucci and Oriola Gjetaj.

CHARLES WALZER (MCMP – LMU), **A labelled sequent calculus for truthmaker logics.**

This paper develops a labelled sequent calculus for logics based on truthmaker semantics, a model-theoretic framework in which the notion of an exact truthmaker is primitive. An exact truthmaker is a state (event, fact-like entity, etc.) that necessitates and is wholly relevant to the truth of a statement. Different kinds of truthmaking, e.g., inexact and loose, can be defined in

terms of exact truthmaking, allowing for the definition of different kinds of entailment relations. Labelled sequent calculi provide a powerful extension of Gentzen-style proof systems, aimed at representing, in a uniform and modular way, model-theoretic notions while preserving important proof-theoretic structural properties such as the syntactic admissibility of cut. Our proof system provides a modular calculus for logics based on inclusive, non-inclusive, and replete classes of exact truthmaker models. The labelling mechanism of our approach also enables the calculus to uniformly represent inexact and loose truthmaking. The difference in label distributions allows for the definition of different kinds of entailment relations for each kind of truthmaking, with a proof of soundness and completeness uniformly covering all such cases. Finally, we provide modal companions for all such logics by defining a suitable labelled calculus for modal information logics.

FRANZISKUS WIESNET (TU Wien, Austria), On the Limits of Recursive Characterizations in the Refined A -Translation.

The refined A -translation, due to Berger, Buchholz, and Schwichtenberg, is based on recursively defined classes of formulas in minimal arithmetic $\mathbf{MA}^\omega$, in particular the classes of definite and goal formulas. One of its basic properties is that $D[\perp := F] \rightarrow D$ is intuitionistically derivable for every definite formula D .

Schwichtenberg and Wainer observed that this property also holds for formulas outside the class of definite formulas, and raised the question of whether there is a useful characterization of all formulas D for which $D[\perp := F] \rightarrow D$ is intuitionistically derivable. Besides definite formulas, the refined A -translation involves three further classes of formulas satisfying related proof-theoretic closure properties. We show that none of the four corresponding classes of formulas admits a recursive characterization.

In addition to this negative result, we extend the framework of the refined A -translation in two directions. First, we add conjunction $\wedge$ to the language of $\mathbf{MA}^\omega$, whose original formulation contains only the logical connectives $\vee$ and $\rightarrow$, and adapt the formula classes accordingly. Second, we prove the corresponding extended version of the refined A -translation theorem and discuss possible recursive extensions of these classes.

QIANGLI ZENG (MCMP – LMU), A Jónsson-Tarski-Goldblatt Representation and Frame Definability for Noncontingency Logic.

In standard normal modal logic, the Jónsson-Tarski duality and the Goldblatt-Thomason theorem rely fundamentally on the normality and additivity of the standard modal operators. The logic of noncontingency, K_Δ , introduces an operator Δ lacking both. The language $\mathcal{L}(\Delta)$ is defined by $\varphi := p \mid \neg\varphi \mid \varphi \wedge \psi \mid \varphi \vee \psi \mid \Delta\varphi$. Semantically, $s \models \Delta\varphi$ iff either $\forall t(sRt \Rightarrow t \models \varphi)$ or $\forall t(sRt \Rightarrow t \not\models \varphi)$. Because the standard duality does not apply directly to this non-normal operator, we establish a custom algebraic representation and a frame definability characterization.

A Noncontingency Algebra (NCA) is a Boolean algebra $(A, +, -, 0, 1)$ with a unary operator Δ satisfying the following identities: $\Delta 1 = 1$, $\Delta x = \Delta(-x)$, $\Delta x \leq \Delta(-x + y) + \Delta(x + y)$, and $\Delta(-y + x) \cdot \Delta(y + x) \leq \Delta x$.

Because Δ does not distribute over meets, we require a contextual necessity operator $L_z(x) := \Delta x \cdot \Delta(-z + x)$. For any ultrafilter $u \in Uf(\mathfrak{A})$ and contingent witness $z \in A$ (where $-\Delta z \in u$), the set $\lambda(u, z) := \{y \in A \mid L_z(y) \in u\}$ forms a proper filter. The contextual necessity operator is an analogous the ‘almost-definable’ schema in the literature.

The canonical ultrafilter frame is $\mathfrak{A}_+ = (Uf(\mathfrak{A}), R_\Delta)$, where $uR_\Delta v$ iff either $\forall x \in A, \Delta x \in u \Rightarrow u = v$, or $\exists x \in A$ such that $-\Delta x \in u$ and $\forall y \in A, L_x(y) \in u \Rightarrow y \in v$. This yields the Representation Lemma: $r(\Delta x) = \Delta_{R_\Delta}(r(x))$, establishing that every NCA is isomorphic to a subalgebra of the complex noncontingency algebra of its canonical frame.

A consequence is a Goldblatt-Thomason type theorem first-order definable Kripke frame class $\mathbb{K}$ is definable in $\mathcal{L}(\Delta)$ if and only if $\mathbb{K}$ is closed under disjoint unions, Δ -generated subframes, Δ -bounded morphic images, and reflects Δ -ultrafilter extensions. Consequently, standard relational properties like reflexivity and transitivity are not definable in $\mathcal{L}(\Delta)$.

ZIXUAN ZHU (Münster), **Rank and independence of imaginaries in pairs of ACF.**

Let T_P be the theory of beautiful pairs of algebraically closed fields of fixed characteristic. It is known that for real tuples in models of T_P , SU-rank coincides with Morley rank and can be computed effectively. Building on Pillay's geometric description (2007) of imaginaries in T_P , we define an additive rank on imaginaries of T_P , called the geometric rank. It takes values in $\omega * \mathbb{N} + \mathbb{Z}$ and coincides with SU-rank on real tuples. It refines SU-rank and characterizes forking in T_P^{eq} , from which we derive an explicit criterion for determining forking independence.